A BROADBAND PARAMETER EXTRACTION TECHNIQUE FOR THE EQUIVALENT CIRCUIT OF PLANAR INDUCTORS

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ABSTRACT

A broadband parameter-extraction technique is developed for the equivalent circuit of planar inductors based on measured S-parameters and the use of Y parameters. The broadband information allow the definition of a concise quality measure and frequency range of application beyond the first resonance. We prove that the negative coupling capacitance appearing in the equivalent circuit is indeed required in order to approximate closely the distributed nature of the inductors even if it appears to contradict the general perception.

I. INTRODUCTION

This paper introduces a new way of extracting the broadband parameters based on the Y-parameter theory for planar inductors. Planar inductors are common matching circuit elements used in GaAs MMIC designs. Its electrical characteristics may be obtained experimentally by measurements[1] or theoretically by static inductance calculations[2,3], coupled transmission formulation[4], and rigorous EM simulations[5,6]. The results are normally presented in terms of scattering parameters. However, from a circuit designer's viewpoint, it is desirable to have a reliable equivalent circuit with a well-defined frequency limit of application.

A traditional way of obtaining the equivalent circuit model is to use an optimization technique in which the circuit element values are systematically varied until the difference between the calculated S-parameters based on the equivalent circuit and the measured S-parameters is minimized over a frequency range. This approach always gives results but does not provide a well-defined quality measure as the acceptance criterion. The results are typically dependent on the frequency range, error functions, error criteria, number of iterations, and initial guess.

In this paper, we present a natural parameter extraction technique using admittance matrix transformation from measured S-parameters. Measurement uncertainties as well as the frequency dependent behaviors are preserved through the transformation, allowing the determination of an acceptance criterion and the applicable frequency range. In addition, due to the accurate on-wafer probe measurement[7], the results actually reveal that the coupling capacitor in the circuit model has a negative value that closely approximates the distributed nature of the line segments. This negative capacitor is theoretically verified.

II. PARAMETER EXTRACTION PROCEDURE

The problem under discussion is a planar inductor and its equivalent circuit model as shown in Figure 1. Based on the EM field considerations, the circuit elements are commonly interpreted as:

L	: inductance, which is the primary element.
R	: resistance, due to the conductor loss.
C1,C2	: capacitance, due to the fringing field to the ground plan.
G1,G2	: conductance, due to the substrate leakage, if any.
Cc	: capacitance, due to the coupling between the lines.

A π network is naturally represented by the admittance matrix as

$$Y12 = Y21 = -j\omega Cc - 1/(R + j\omega L) \qquad (1)$$

$$Y11 = G1 + j\omega C1 - Y12 \qquad (2)$$

$$Y22 = G2 + j\omega C2 - Y12 \qquad (3)$$

If we extract the circuit parameters from measured S-parameters, the procedure begins with the transformation of the scattering matrix into admittance matrix. Then using equations (1) and (2), we immediately obtain C1 and G1, and similarly C2, and G2 from (1) and (3), at every measurement frequency. However, the information in Y12 does not allow a unique determination of R, L, and Cc because there are three unknowns with only two equations. One additional piece of information is required. Since the purpose of the lumped equivalent circuit is to approximate the electrical behavior for as broadband as possible, we make the following assumptions:

[1] Cc is a constant independent of frequency. This assumption is acceptable because its value is small and has secondary effect on the frequency behavior.

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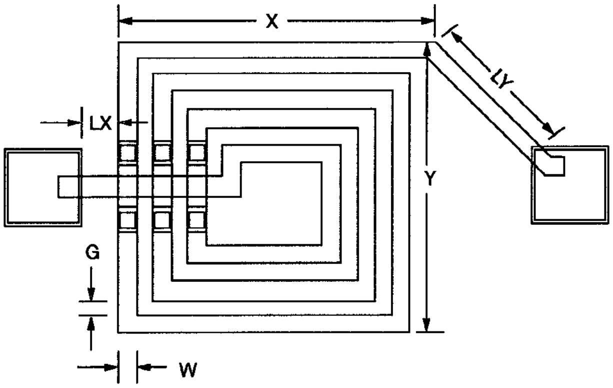
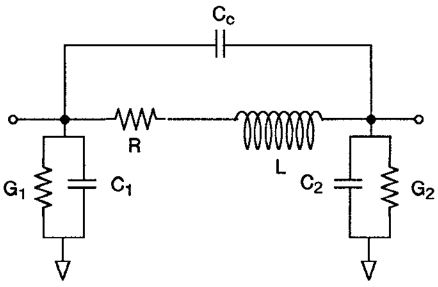


(a) A PLANAR SPIRAL INDUCTOR



(b) ITS EQUIVALENT CIRCUIT

Figure 1 Layout and equivalent circuit of a rectangular spiral inductor.

[2] The value of Cc should be adjusted so that the primary element of inductance remains approximately constant to as high frequency as possible.

With this assumptions and equation (1), we can uniquely determine the rest of the equivalent circuit elements. The frequency limits of its application can also be determined by a given set of acceptance error criteria.

III. EXAMPLES

To validate the parameter extraction procedure, we have studied the characteristics of a rectangular spiral inductor defined in a common CAD package, Supercompact. A three-turn inductor on a 100-μm thick GaAs substrate is defined as follows:

RECI 1 2 LI = 10 μm AI = 90 μm BI = 90 μm N = 3
W = 30 μm S = 15 μm GaAs

The structure has a low-frequency inductance of 2.07 nH and a resonance at 14.1 GHz. The S-parameters generated by Supercompact are then used in the parameter-extraction procedure described above. The results are summarized in Figure 2, showing that the equivalent circuit can approximate the spiral inductor closely up to 14 GHz with less than 4% deviation in the inductance value. Notice that the coupling capacitance is negative, i.e., Cc = –0.027 pF. Validity of this negative nature will be theoretically justified as explained shortly.

We have also applied this procedure to extract the equivalent-circuit parameters from measured S-parameters for a set of microstrip rectangular spiral inductors. The results are summarized in Table 1. We fabricated the inductors on a

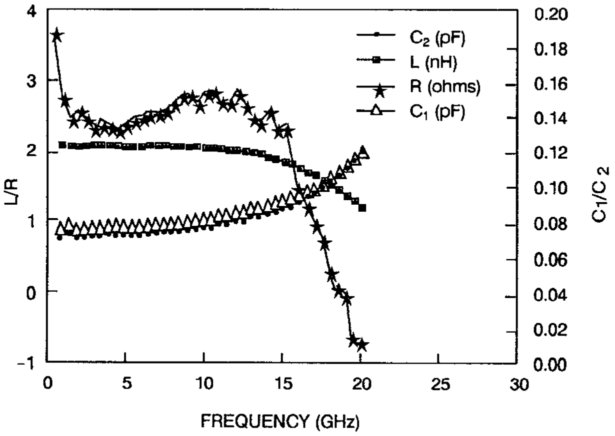


Figure 2 Equivalent circuit values of a microstrip spiral inductor as a function of frequency.

TABLE 1
EQUIVALENT CIRCUIT VALUES OF MIRCOSTRIP SPIRAL INDUCTORS

W (μm)	G (μm)	X (μm)	Y (μm)	N	LX (μm)	LY (μm)	L (nH)	R (ohm)	C1 (pf)	C2 (pf)	Cc (pf)	Fu (GHz)	Fo (GHz)
20	15	100	100	1.25	225	225	0.47	1.245	0.036	0.037	–0.018	>18*	>18*
20	15	200	200	2.25	175	200	1.11	2.62	0.059	0.066	–0.015	>18*	>18*
20	15	300	300	3.25	85	200	2.64	4.47	0.087	0.119	–0.017	14	10.5
20	15	350	350	4.25	85	140	4.09	6.53	0.102	0.141	–0.017	10	7.75

*Measured data is up to 18 GHz only.

100-µm thick GaAs substrate and carried out the measurements using accurate on-wafer probing procedures[7]. The results in Table 1 show that the spiral inductors can be represented by the lumped equivalent up to an upper frequency limit, Fu (4% deviation in inductance) circuits beyond their first resonant frequency, Fo. The broadband representation is partially due to the negative value of the coupling capacitance.

IV. DISCUSSIONS

Is the negative coupling capacitance an artificial factor accidentally introduced to increase the frequency range of application, or is it a natural factor required to approximate more closely the distributed nature of the spiral structure? To answer this question, let us examine a special case of a section of transmission line being the inductor. We assume an ideal lossless case for simplicity. The inductor is to be represented by a π network as shown in Figure 3(b). Because of the structural symmetry, we can apply the concept of even- and odd-excitations from both terminals to the formulation. For even-excitation, we place an open circuit at the center plane of symmetry. Therefore,

$$Yin(EVEN) = jY\tan(\beta\ell/2) = Ya = Yb \qquad (4)$$

where β is the phase constant and ℓ the length of the transmission line. We can see that Ya and Yb are simply open stubs which are capacitive at low frequencies. Similarly, for odd-excitation, we place a short circuit at the plane of symmetry. We obtain

$$Yin(ODD) = -j(Y/2)\cot(\beta\ell/2) = Ya + 2Yab \qquad (5)$$

Subtract eqn.(4) from eqn.(5), we obtain

$$Yab = -j(Y/2)\cot(\beta\ell/2) + j(-Y/2)\tan(\beta\ell/2) \qquad (6)$$

which is a short-circuited stub in parallel with an open-circuited stub having a negative admittance! If we take a lumped element approximation ($\beta\ell \geq 0$) for eqns. (4) and (6), we get

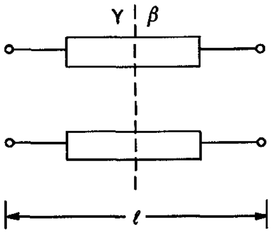
$$Ya = Yb = j(Y\beta\ell/2) = j\omega(C\ell/2) \qquad (7)$$

$$Yab = 1/(j\omega L\ell) + j\omega(-C\ell/4) \qquad (8)$$

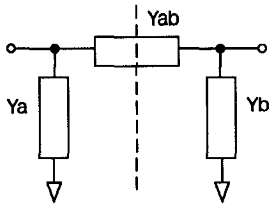
where C and L are the distributed capacitance and inductance of the transmission line. The lumped equivalent circuit is shown in Figure 3(c). It is now clear that the negative capacitance coupling across the inductor is a natural result representing the distributed components.

V. CONCLUSIONS

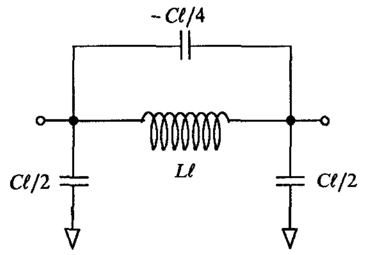
We have developed a broadband parameter-extraction technique for the equivalent circuit of planar inductors based on measured S-parameters. The admittance matrix technique preserves all the information through the transformation including the frequency-dependent behavior and measurement uncertainties. It computes the value of the equivalent circuit at every frequency point of measurement. This allows us to define a concise quality measure and a practical frequency range of application beyond the first resonance. Furthermore, we found that the coupling capacitance is a negative value which is required to approximate closely the distributed nature of the spiral inductors, even though such a negative value appears to contradict the general perception.



(a) A SECTION OF TRANSMISSION LINE



(b) ITS π NETWORK REPRESENTATION



(c) ITS LUMPED EQUIVALENT CIRCUIT

Figure 3 Lumped model of distributed transmission line.

The broadband parameter-extraction technique can also be applied to other passive components such as capacitors and resistors, or even active devices.

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